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Squeezed Fermions at Relativistic Heavy Ion Collider

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Abstract

Large back-to-back correlations of observable fermion – anti-fermion pairs are predicted to appear, if the mass of the fermions is modified in a thermalized medium. The back-to-back correlations of protons and anti-protons are experimentally observable in ultra-relativistic heavy ion collisions, similarly to the Andreev reflection of electrons off the boundary of a superconductor. Surprisingly, the fermionic back-to-back correlations are similar to the bosonic back-to-back correlations.

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Introduction.— In high energy heavy ion collisions, hot and dense hadronic matter is expected to be created in conditions similar to the ones in the early Universe about a few μsec after the Big Bang. Under these conditions, strongly interacting particles may propagate with a mass that differs from the mass in the asymptotic vacuum. Recently, it has been discovered that in-medium mass-modifications induce large back-to-back correlations (BBC) among pairs of asymptotic, observable bosons [1,2]. However, it was not known before if the bosonic BBC would have an analogous effect in the fermion sector. In this Letter we show that fermion - anti-fermion pairs also exhibit BBC, if they propagate with a modified mass in the medium.

Quantum statistics enhances the probability of observing pairs of bosons in similar momentum states, while it suppresses the probability of observing pairs of fermions with nearby momenta. The resulting Bose-Einstein or Fermi-Dirac correlations can be explored to reconstruct the space-time picture of the particle emitting sources. Surprisingly, we find that the dominant term of the fermionic BBC (fBBC) is a positive correlation, and similar in strength to the bosonic BBC of refs. [1,2]. The fBBC could be observed experimentally in ^{197}Au + ^{197}Au collisions with $\sqrt{s}$ = 200 AGeV at the Relativistic Heavy Ion Collider (RHIC), scheduled to start taking data at the Brookhaven National Laboratory in the year 2000.

The bosonic BBC (bBBC) have a quantum optical analogy, namely the correlations in thermalized ensembles of two-mode squeezed states. It turns out that fBBC also have an analogy, which is the Andreev reflection, well-known in solid state physics. It refers to the scattering of electrons off the boundary of a superconductor - normal conductor junction. The reflected electrons are used to study the properties of the superconductors [3]. Our results for fBBC generalize Andreev's reflection to the case, when superconductivity is suddenly switched off in the whole volume of the material, so that a junction prevails not at a given position for a long time but at a given instant, in the whole medium.

Basic assumptions.— We assume the validity of the concepts of local thermalization, hydrodynamics, and a short duration of particle emission. These concepts are in agreement with the observable single-particle spectra and two-particle correlations of pions, kaons and protons [4]. We also assume the validity of the effective Hamiltonian

$$H = H_0 + H_I, \tag{1}$$

where

$$H_0 = \int d\mathbf{x} \ : \bar{\psi}(\mathbf{x})(-i\boldsymbol{\gamma} \cdot \boldsymbol{\nabla} + M)\psi(\mathbf{x}) : \tag{2}$$

is the Hamiltonian in vacuum, and

$$H_I = -\int d\mathbf{x}d\mathbf{y}\, \Sigma(\mathbf{x} - \mathbf{y}) \ : \bar{\psi}(\mathbf{x})\psi(\mathbf{y}) : \tag{3}$$

describes the medium modifications. In the above, M is the value of the proton (or in general, baryon) mass in the asymptotic vacuum, ψ and $\bar{\psi}$ are the fermion field operators which satisfy the usual equal-time anti-commutation relations. The self-energy term Σ can be determined from a detailed calculation based on, for example, the Walecka or the Zimányi - Moszkowski models [5] of hot and dense nuclear matter.

For a Dirac particle under the influence of mean fields in a many-body system, one can write its self-energy as

$$\Sigma = \Sigma^s + \gamma^0\Sigma^0 + \gamma^i\Sigma^i. \tag{4}$$

We denote the scalar component of the self-energy as $\Sigma^s = \Delta M$. In all model calculations that we know of, Σ^0 is weakly momentum dependent and Σ^i is very small. Therefore, in the following we take a constant value for Σ^0, and neglect Σ^i. For a locally thermalized nuclear matter, the role of Σ^0 is to shift the chemical potential. See the lines around Eq. (23) of ref. [6] that imply

$$\mu_* = \mu - \Sigma^0. \tag{5}$$

As we present our results as a function of the net baryon density, the value of Σ^0 (or the difference between μ and μ_*) needs not be specified.

In momentum space, the effective Hamiltonian of eq. (1) thus corresponds to a momentum-dependent in-medium mass, $M_*(\mathbf{k}) = M - \Delta M(|\mathbf{k}|)$.

The following symbolic notation is introduced to simplify the presentation: $F_1 \equiv F(\mathbf{k}_1, t_1, \lambda_1)$ stands for any function F, that depends on the momentum $\mathbf{k}_1$, the particle type t_1 and the spin index λ_1. The type of the particle can be $t_1 = \mathrm{f}$ (fermion) or $t_1 = \mathrm{a}$ (anti-fermion). The notation F_{-1} stands for the same function F with opposite momentum $\mathbf{k}_{-1} \equiv -\mathbf{k}_1$, opposite spin and particle type.

The baryon field ψ at time $t = 0$ can be expanded as

$$\psi(\mathbf{x}) = \frac{1}{\sqrt{V}} \sum_1 \left(u_1 a_1 + v_{-1} a^\dagger_{-1} \right) e^{i\mathbf{k}_1 \cdot \mathbf{x}}, \tag{6}$$

where V is the volume of the system, u_1 and v_{-1} are Dirac spinors and the summation extends over momenta $\mathbf{k}_1$ and spin projection $\lambda_1 = 1/2, -1/2$. The operator a_1 annihilates a fermion with momentum $\mathbf{k}_1$ and spin projection λ_1, while the operator $a^\dagger_{-1}$ creates an anti-fermion with momentum $-\mathbf{k}_1$ and with spin projection λ_{-1}. The fermion creation and annihilation operators satisfy canonical anti-commutation relations; they are also denoted here by a as in the bosonic case [1,2] to highlight, as much as possible, the similarities between these cases.

Spectra and correlations for mass-shifted fermions.— The single-particle and the two-particle invariant momentum distributions can be summarized using the symbolic notation including particles and anti-particles simultaneously:

$$N_1(1) = \omega_1 \langle a^\dagger_1 a_1 \rangle, \tag{7}$$

$$N_2(1,2) = \omega_1 \omega_2 \langle a^\dagger_1 a^\dagger_2 a_2 a_1 \rangle, \tag{8}$$

$$\langle a^\dagger_1 a^\dagger_2 a_2 a_1 \rangle = \langle a^\dagger_1 a_1 \rangle \langle a^\dagger_2 a_2 \rangle - \langle a^\dagger_1 a_2 \rangle \langle a^\dagger_2 a_1 \rangle + \langle a^\dagger_1 a^\dagger_2 \rangle \langle a_2 a_1 \rangle.$$

In the above, $\langle \hat{O} \rangle$ denotes the expectation value of operator $\hat{O}$ in the thermalized medium, to be determined for example in the formalism of thermo field dynamics [7].

The chaotic and squeezed amplitudes are

$$G_c(1,2) = \sqrt{\omega_1\omega_2}\langle a_1^\dagger a_2\rangle, \tag{9}$$

$$G_s(1,2) = \sqrt{\omega_1\omega_2}\langle a_1 a_2\rangle. \tag{10}$$

The chaotic amplitude, $G_c(1,2) \equiv G(1,2)$ is non-vanishing for identical type of fermions, $t_1 = t_2$, while the squeezed amplitude $G_s(1,2)$ is non-vanishing for particle - anti-particle pairs, $(t_1, t_2) = (\mathrm{f}, \mathrm{a})$ or (a, f). The squeezed $G_s(1,2)$ is non-negligible if $\Delta M(|\mathbf{k}|) \neq 0$.

The two-particle correlation function is defined as

$$C_2(\mathbf{k}_1, \mathbf{k}_2) = \frac{N_2(\mathbf{k}_1, \mathbf{k}_2)}{N_1(\mathbf{k}_1)N_1(\mathbf{k}_2)} \ . \tag{11}$$

Results for a homogeneous system.— In order to evaluate the observable effects of in-medium mass-modification of fermions, one has to describe the process of particle freeze-out in a quantum-mechanical manner. Let us introduce the annihilation operator b_1 for the in-medium quanta, and define the annihilation operator a_1 for the asymptotic field. While it is the a-quanta that are observed, it is the b-quanta that are thermalized in medium. The in-medium thermal density matrix is diagonal in terms of the b quanta, while the asymptotic Hamiltonian is diagonal in terms of the a quanta. Assuming a sudden freeze-out, we find that the asymptotic and the in-medium creation and annihilation operators are related by a *fermionic Bogoliubov-Valatin transformation*

$$\begin{pmatrix} a_1 \\ a_{-1}^\dagger \end{pmatrix} = \begin{pmatrix} c_1 & \frac{r_1}{|r_1|}\, s_1\, A \\ -\frac{r_1^*}{|r_1|}\, s_1^*\, A^\dagger & c_1^* \end{pmatrix} \begin{pmatrix} b_1 \\ b_{-1}^\dagger \end{pmatrix}, \tag{12}$$

where $c_1 = \cos r_1$, $s_1 = \sin r_1$, and

$$\tan(2r_1) = -\frac{|\mathbf{k}_1|\Delta M(\mathbf{k}_1)}{\omega(\mathbf{k}_1)^2 - M\Delta M(\mathbf{k}_1)} \tag{13}$$

is the fermionic squeezing parameter. In Eq. (12) A is a 2×2 matrix with elements $A_{\lambda_1\lambda_2} = \chi_{\lambda_1}^\dagger \sigma \cdot \hat{\mathbf{k}}_1 \tilde{\chi}_{\lambda_2}$, where $\hat{\mathbf{k}}_1 = \mathbf{k}_1/|\mathbf{k}_1|$, χ is a Pauli spinor and $\tilde{\chi} = -i\sigma^2\chi$. Since r is real in the present case, we will drop the complex-conjugate notation in what follows.

If a thermal gas of b fermions freezes out suddenly at temperature T, the observed single a-particle spectrum is given by

$$N_1(\mathbf{k}_1) = \frac{V}{(2\pi)^3}\,\omega_1\, n_1(\mathbf{k}_1), \tag{14}$$

$$n_1(\mathbf{k}_1) = c_1^2\, n_1 + s_{-1}^2(1 - n_{-1}), \tag{15}$$

$$n_{\pm 1} = \frac{1}{\exp\left[(\Omega_{\pm 1} - \mu_{\pm 1})/T\right] + 1}, \tag{16}$$

where $\mu_{\pm 1} = \pm\mu_1$ and

$$\Omega_1 = \sqrt{\mathbf{k}_1^2 + [M - \Delta M(|\mathbf{k}_1|)]^2}. \tag{17}$$

Note that the single-particle spectrum includes a *squeezed vacuum* contribution. While the thermal part of the spectrum falls off exponentially for large values of $|\mathbf{k}|$, the squeezed vacuum contribution falls off only as a power of $|\mathbf{k}|$, because of the term s_{-1}^2 in Eq. (15).

For such a homogeneous system, the net baryon density ρ_B, the chaotic, and the squeezed amplitudes can be easily calculated using e.g. thermofield dynamics [7]. They are given by

$$\rho_B = \frac{g}{V}\sum_{\mathbf{k}_1}\,(n_1 - n_{-1}), \tag{18}$$

$$G_c(1,2) = \frac{V}{(2\pi)^3}\,[c_1^2\, n_1 + s_{-1}^2\,(1 - n_{-1})]\,\delta_{1,2}, \tag{19}$$

$$G_s(1,2) = \frac{V}{(2\pi)^3}\,[(1 - n_1 - n_{-1})c_1 s_{-1}\,A^\dagger]\,\delta_{1,-2}, \tag{20}$$

Notation $\delta_{1,-2}$ implies that the squeezed amplitude is non-vanishing only for fermion anti-fermion pairs.

There are two kinds of fermionic two-particle correlation functions that are summarized by Eq. (11) . Similarly to the bosonic case, let us denote by $C_2^{(++)}(\mathbf{k}_1, \mathbf{k}_2)$ the case when the two particles are identical fermions, and by $C_2^{(+-)}(\mathbf{k}_1, \mathbf{k}_2)$ when particle 1 is a fermion and 2 is an anti-fermion. The correlation functions $C^{(--)}(\mathbf{k}_1, \mathbf{k}_2)$ and $C^{(-+)}(\mathbf{k}_1, \mathbf{k}_2)$ can be obtained from the $(++)$ and $(+-)$ correlations by a trivial exchange of particle and anti-particle labels. For an infinite, homogeneous thermalized medium, these correlation functions are non-trivial only for identical or back-to-back momenta, $\mathbf{k}_2 = \pm\mathbf{k}_1$. We find that $C_2^{(++)}(\mathbf{k}, \mathbf{k}) = C_2^{(--)}(\mathbf{k}, \mathbf{k}) = 0$, the canonical value of the Fermi-Dirac correlation function, reflecting the anti-correlation of identical fermions, due to the Pauli exclusion principle.

The BBC for fermion – anti-fermion pairs reads as

$$C_2^{(+-)}(\mathbf{k}_1, -\mathbf{k}_1) = 1 + \frac{|G_s^{(+-)}(1,-1)|^2}{G_c^{(++)}(1,1)G_c^{(--)}(-1,-1)} =$$
$$1 + \frac{(1-n_1-n_{-1})^2(c_1 s_{-1})^2}{\left[c_1^2 n_1 + s_{-1}^2(1-n_{-1})\right]\left[c_{-1}^2 n_{-1} + s_1^2(1-n_1)\right]}. \tag{21}$$

From this equation it follows that the fermionic BBC are unlimited from above. For sufficiently large values of $|\mathbf{k}_1|$, the Fermi-Dirac distribution n_1 falls exponentially, while s_1 decreases only as a power-law. Hence, for sufficiently large values of $|\mathbf{k}_1|$, the fBBC diverge similarly to the case of bosonic BBC, as $C^{(+-)}(\mathbf{k}_1, -\mathbf{k}_1) \propto 1 + 1/s_1^2 \to \infty$. This divergence happens for small values of the mass-shift and for large values of $\mathbf{k}_1$ in both the fermionic and the bosonic cases.

In Fig. 1 we show the fBBC for $p\bar{p}$ pairs as a function of the in-medium mass $M_* = M - \Delta M$, for three illustrative values of the *net* baryonic density ρ_B: the normal nuclear matter density, one tenth of the nuclear matter density, and $\rho_B = 0$. This last value of ρ_B corresponds to a baryon-free region, as expected to be formed at RHIC. We also show in this figure the corresponding result for bBBC of ϕ mesons. A finite time suppresion factor was applied, following the example of ref. [2]. We observe that the fBBC are strongly enhanced when the net baryonic density decreases, and that the shape of fBBC for $\rho_B = 0$ is rather similar to bBBC. Thus, one might say that BBC for a baryon free region are approximately supersymmetric, since their strength and shape are approximately independent of the bosonic or the fermionic nature of the particles. Surprisingly enough, fBBC are not only positive, similarly to the bosonic case, but they are also of the same order of magnitude as the corresponding bBBC.

The expected magnitude of the effect is illustrated in Fig. 2 for two, typical momenta of thermal-looking proton spectrum at CERN SPS Pb + Pb collisions [8], for $|\mathbf{k}| = 500$ MeV and for $|\mathbf{k}| = 800$ MeV. As in Fig. 1, a finite time suppression factor is applied, with $\Delta t = 2$ fm/c. Fig. 2 indicates that the fBBC is strongly enhanced for increasing momentum $|\mathbf{k}|$. Fig. 2 also highlights that the magnitude of the fBBC is greatly enhanced as the net baryon density decreases from normal nuclear density to a vanishing value.

BBC for bosons and fermions.— It is particularly interesting to note that Eq. (21), together with Eq. (19) of ref. [2], can be casted into a unified form, namely

$$C_2^{(+-)}(\mathbf{k}_1, -\mathbf{k}_1) = 1 + \frac{(1 \pm n_1 \pm n_{-1})^2 (c_1 s_{-1})^2}{[c_1^2 n_1 + s_{-1}^2 (1 \pm n_{-1})]\,[c_{-1}^2 n_{-1} + s_1^2 (1 \pm n_1)]} \tag{22}$$

$$n_1 = \frac{1}{\exp[(\Omega_1 - \mu_1)/T] \mp 1}, \tag{23}$$

where the upper sign stands for bosons, and the lower sign for fermions. Note, however, that the symbols $s_{\pm 1}$ and $c_{\pm 1}$ are related to $\sin(r_{\pm 1})$ and $\cos(r_{\pm 1})$, as defined by Eq.(13) in the fermionic case, while in the bosonic case, these symbols mean $s_{\pm 1} = \sinh(r_{\pm 1})$ and $c_{\pm 1} = \cosh(r_{\pm 1})$, where $r_{\pm 1}$ is also real but given by Eq. (11) of ref. [2]. It follows that the BBC diverge with increasing $|\mathbf{k}_1|$ as the inverse of the single-particle spectra, both in the fermionic and bosonic cases.

Summary.— BBC stands for Back-to-Back Correlations of particle and anti-particle pairs. These correlations appear if in-medium interactions lead to the modification of hadronic masses in the medium. The origin of these back-to-back correlations is an entirely quantum effect, related to the propagation of particle fields through a space-like boundary surface between the medium and the asymptotic vacuum. The strength of these correlations can be unlimitedly large, and the shape of the BBC is similar for fermions and for bosons. Deep mathematical and physical reasons are behind these similarities. A sudden freeze-out of thermalized medium-modified quanta to asymptotic fields is described by a bosonic or a fermionic Bogoliubov transformation. Although these transformations are canonical, they connect Fock spaces that become unitarily inequivalent in the infinite volume limit [7] and the vacuum state of the medium corresponds to strongly correlated, squeezed particle – anti-particle states of the asymptotic quanta.

At large values of momenta, the similarity of fermionic BBC to bosonic BBC reflects the supersymmetry of the decaying vacuum of the medium to boson - anti-boson and to fermion - anti-fermion pairs. This effect is non-perturbative in terms of the in-medium mass-shifts, as it vanishes in case of exactly zero in-medium mass-modification. However, for large values of $|\mathbf{k}|$ and small values of mass-shifts, BBC can be very large and should be observable.

In this Letter, we have extended the concept of back-to-back correlations to an important new domain of broad experimental and theoretical interest. As a reduction of the net baryon density by a factor of 10 increases dramatically the magnitude of the effect, the almost baryon-free mid-rapidity region of $Au + Au$ collisions at RHIC seems to be an ideal place to find the back-to-back correlations of proton - anti-proton or Λ - $\overline{\Lambda}$ pairs experimentally.

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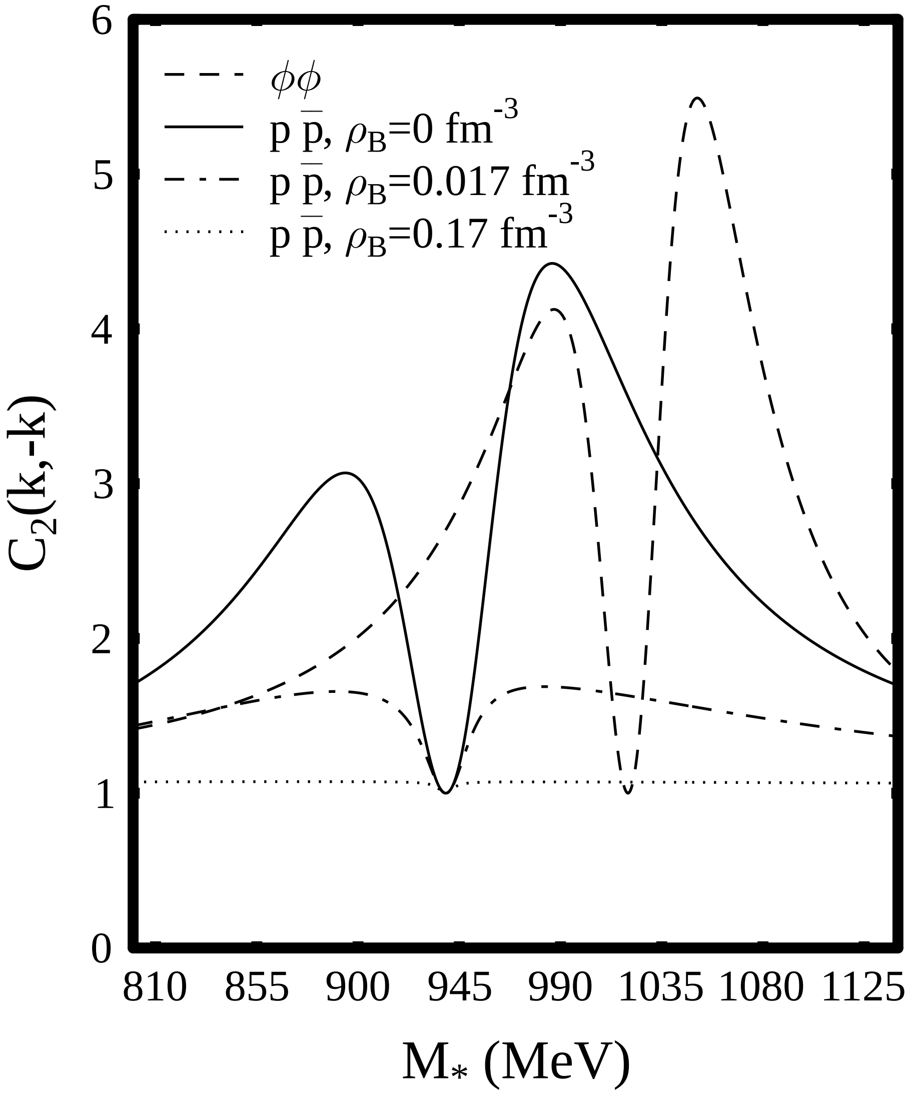


FIG. 1. Back-to-back correlations of proton - anti-proton pairs and ϕ-meson pairs, for $T = 140$ MeV, $\Delta t = 2$ fm/c and $|\mathbf{k}| = 800$ MeV/c.

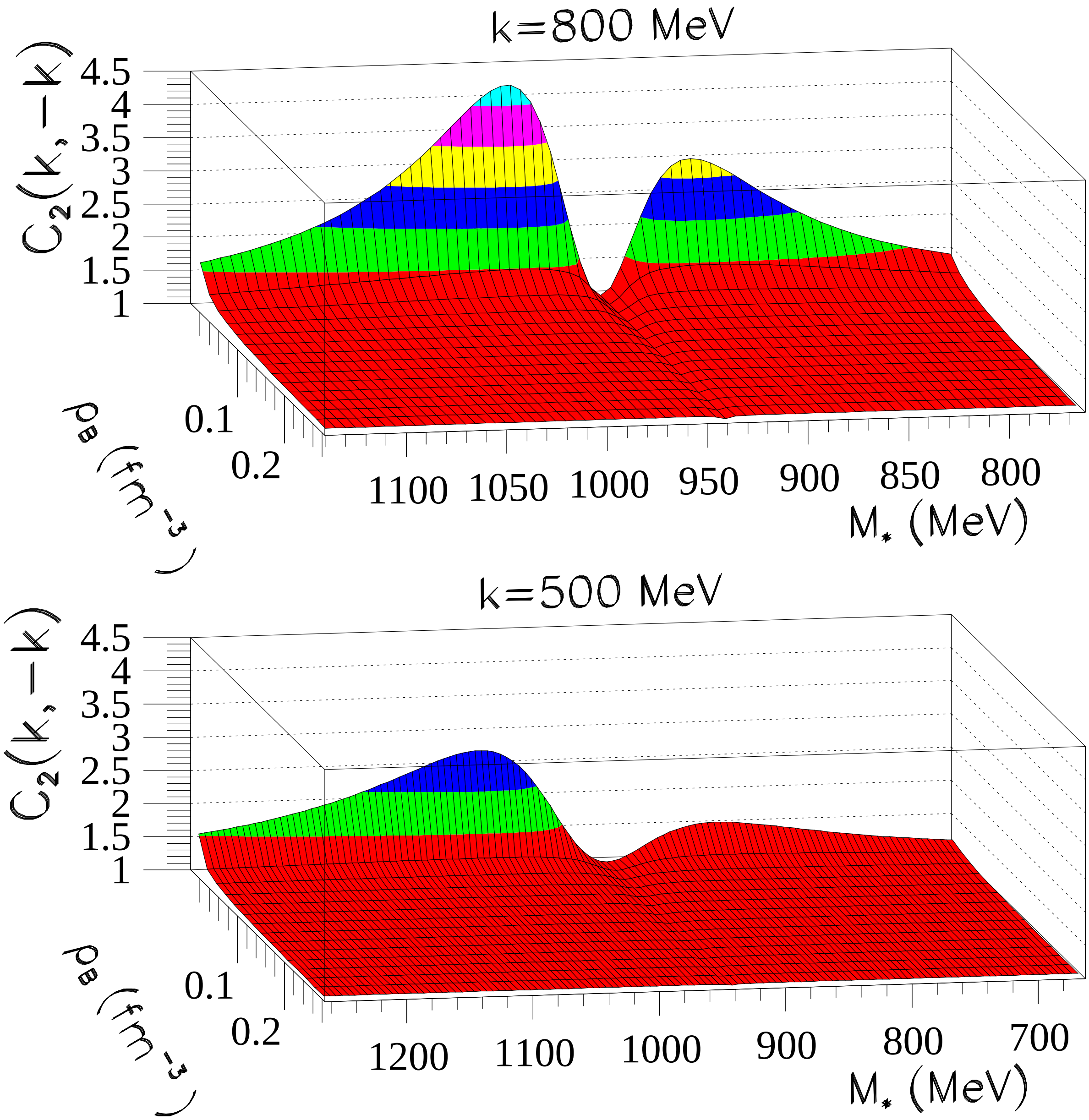
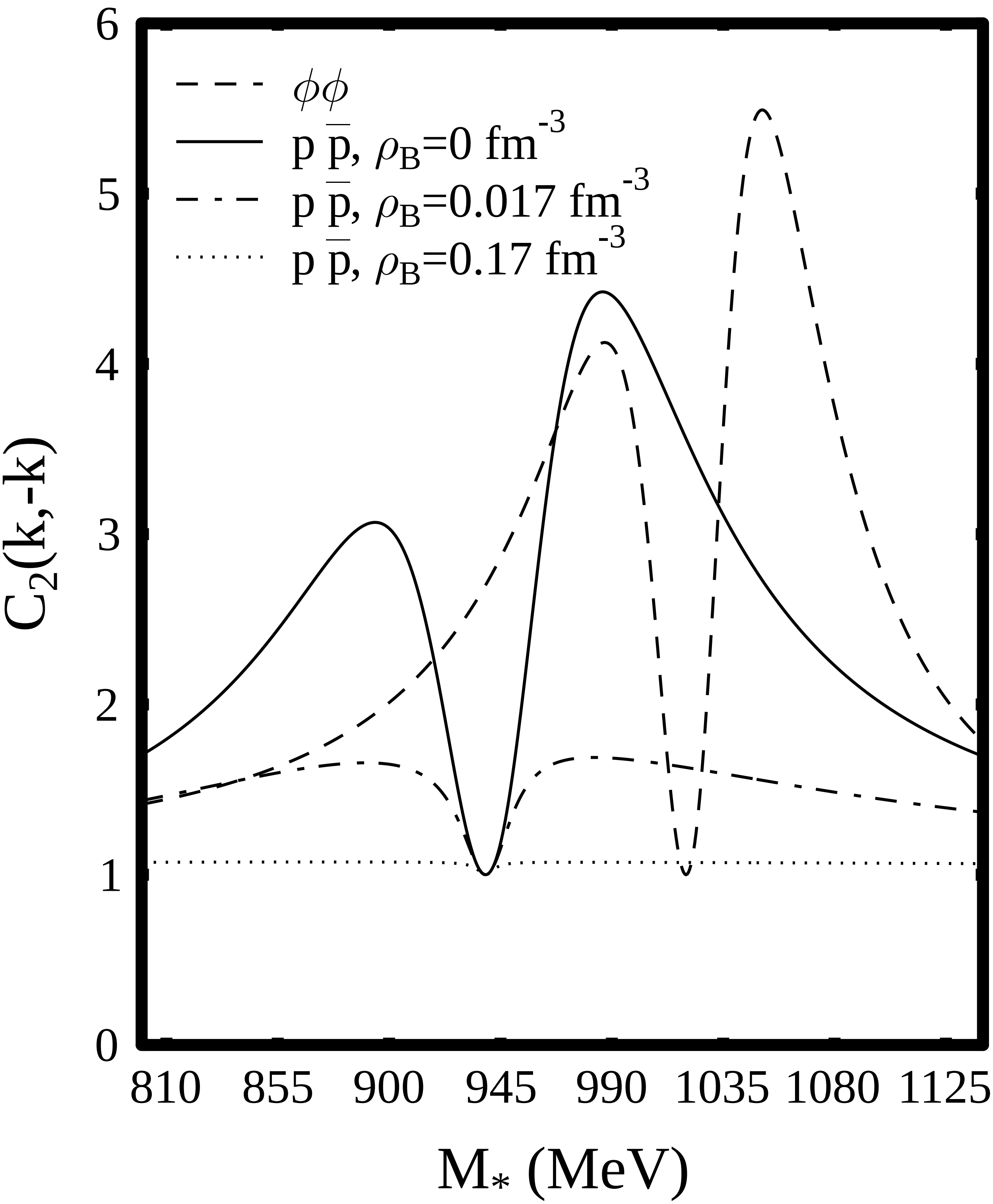
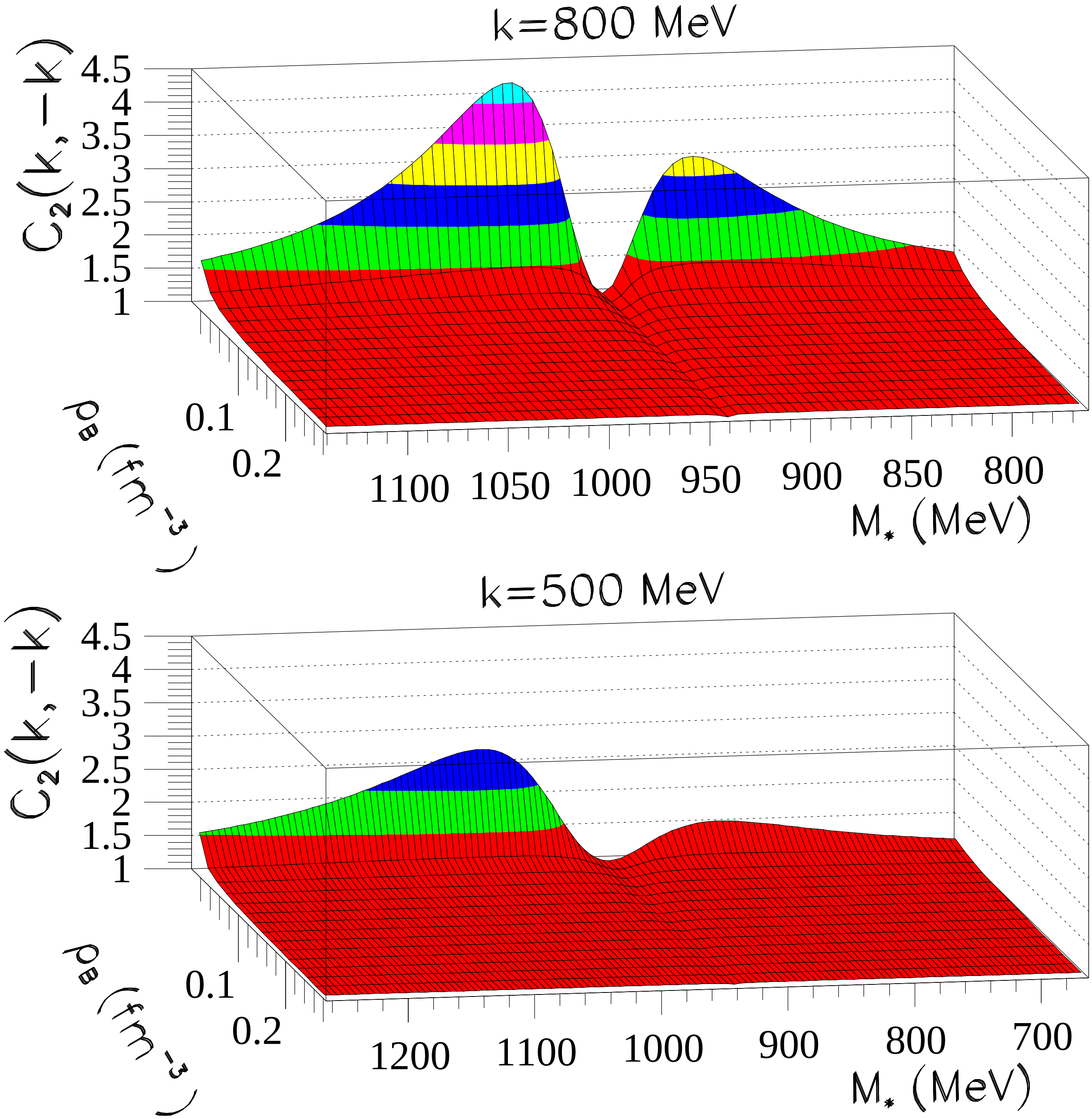


FIG. 2. Dependence of the fBBC on the in-medium modified proton mass, M_*, and on the net baryon density, ρ_B, for $T = 140$ MeV, $\Delta t = 2$ fm/c and two typical values of the momentum $|\mathbf{k}|$. The fBBC strong increases with decreasing net baryon density and with increasing values of $|\mathbf{k}|$.



$\phi\phi$
p $\bar{p}$, ρ_B=0 fm^{-3}
p $\bar{p}$, ρ_B=0.017 fm^{-3}
p $\bar{p}$, ρ_B=0.17 fm^{-3}
C_2(k,-k)
0
1
2
3
4
5
6
810
855
900
945
990
1035
1080
1125
M_* (MeV)


k=800 MeV
$C_2(k,-k)$
4.5
4
3.5
3
2.5
2
1.5
1
0.1
0.2
ρ_B (fm^{-3})
1100
1050
1000
950
900
850
800
M_* (MeV)
k=500 MeV
$C_2(k,-k)$
4.5
4
3.5
3
2.5
2
1.5
1
0.1
0.2
ρ_B (fm^{-3})
1200
1100
1000
900
800
700
M_* (MeV)